

## Using the Chinese Abacus

This is a work-along exercise. If you have an abacus, use it to carry out the examples. If you don't have an abacus at hand, use the simulated abacus found at:

[http://matti.usu.edu/nlvm/nav/frames\\_asid\\_196\\_g\\_3\\_t\\_1.html](http://matti.usu.edu/nlvm/nav/frames_asid_196_g_3_t_1.html)



**A Chinese abacus in the “home” position**

The abacus can be a very practical computational device. It is still used to this day for check-out stand calculations in small retail shops throughout the orient. Learning to use an abacus is a worthwhile exercise in cultural appreciation, but the main reason for this exercise is mathematical. Since the abacus represents numbers in a very concrete form, it has great value in introducing the number system and simple arithmetic to young students.

Our primary purpose is to make whole numbers and the base-10 place value system more concrete for younger students.

We represent the number “three” with an abstract symbol, “3”, but a young child will better understand concept of “three-ness” when it is represented by three counters. The advantage of direct representation soon breaks down, however. The number 46 is not so well represented by 46 counters. With the exception of certain autistic savants (like the character in “Rain Man”) most people can't instantly recognize 46-ness at a glance as they can recognize 3-ness. Instead, we typically group single items into piles and count the piles. Thus, for example, a \$5 bill represents five \$1 bills and a \$10 bill represents two \$5 's or ten \$1's. People would be annoyed if we paid \$46 by laying out 46 ones. Instead we would typically lay out four \$10's, a \$5, and a \$1. The value of that combination typically *can* be recognized at a glance, at least by adults.

This is the basis of the abacus.

The Chinese abacus has many columns of beads divided into upper and lower sections by a horizontal central bar. Each column has five beads below the bar and two beads above the bar.

To begin, tip the abacus up, so all the lower beads move away from the center. Then lay it flat and move the upper beads away from the bar by running the back of your finger between the top beads and the bar. When all the beads are moved away from the bar the abacus is in the “home” position representing “zero”. (See the first illustration above.)

The columns of the abacus represent place value in the base 10 system. To work with whole numbers, the right-most column is the 1’s column. The next column to the left is the 10’s column, followed by the 100’s and 1000’s columns, etc.



**The number being displayed here is 1538**

In any column, one bead in the lower section is worth one unit of that column’s place value. We will call these 1-beads. Thus a 1-bead in the 1’s column is worth 1. A 1-bead in the 100’s column is worth 100, etc. The upper beads are worth five 1-beads in the same column. We will call these 5-beads.

Start counting by bringing 1-beads in the right-most column up to the center bar. You can count as high as 5 units in any column by moving the 1-beads up. When you reach 5, you can “trade in” the five 1-beads for one 5-bead in the same column. After 5, continue counting by raising 1-beads. Six is represented by a combination of a 5-bead and a 1-bead; seven is represented by a combination of a 5 bead and two 1-beads; etc. Ten *could* be represented by a 5-bead and five 1-beads, but the five 1-beads trade in for a second 5-bead, and the two 5-beads trade in for a single 1-bead in the next column.

*Advanced users never actually use the 5<sup>th</sup> lower bead or the 2<sup>nd</sup> upper bead. Instead of sliding the extra bead, then immediately undoing the move to trade it in, they eliminate the intermediate steps. After counting to 4, the next count is to slide down the 5-bead and remove the four 1-beads. After counting to 9, the next count is to remove all the beads in that column and add a 1-bead in the next column. The Japanese version of the abacus (called a soroban) forces this more efficient method because it has only 4 beads below the bar and one above the bar. However, as we shall see later, the extra beads simplify some of the more difficult maneuvers down the road.*

## **Adding**

Adding is accomplished by setting a number on the abacus, then setting enough additional beads in each column to represent the second number. For small digits (where carrying is not involved) this is straightforward. Try the following examples.<sup>1</sup>

$$1+2, 1+3, 2+2, 5+1, 6+3, 7+2$$

When you are done, you should recognize the result by counting up the beads. The same method applies to multi-digit numbers.

Try these:

$$35+14, 53+21$$

If the second number has digits greater than 5, you may need to slide two beads to do the addition. Try these:

$$1+7, 3+6, 2+7, 1+8$$

When two numbers smaller than 5 are added to get a number larger than 5 you need to do the equivalent of “making change”. For example try  $3+3$ .

Step 1: Set three 1-beads.

Step 2: Since there aren't three more 1-beads, set a 5-bead and take away two 1-beads. (This is like paying for a \$3 item with a \$5 bill and getting \$2 change.)

Try these:

$$3+4, 4+4, 2+4$$

When numbers go over 10 in any column, trade-ins will involve the next column. Consider  $2 + 8$ , for example.

Step 1: Set two 1-beads.

Step 2: For the 8, you could add a 5-bead and three more 1-beads.

Step 3: Trade in the five 1-beads, for the second 5-bead.

Step 4: Trade in the two 5-beads for a single 1-bead in the next column.

This is the most concrete way to do the problem, but an advanced user would do this instead:

Step 1: Set two 1-beads.

Step 2: Recognize that “8” is “10-2”, so add a 1-bead in the second column and take away two 1-beads in the current column.

One of the benefits of working with an abacus is the constant practice representing numbers from 6 through 9 in terms of their “ $5 + \underline{\quad}$ ” and “ $10 - \underline{\quad}$ ” equivalents.

$$6 = 5 + 1 = 10 - 4$$

$$7 = 5 + 2 = 10 - 3$$

$$8 = 5 + 3 = 10 - 2$$

$$9 = 5 + 4 = 10 - 1$$

Try these:

$$3 + 8, 4 + 9, 4 + 7, 4 + 6$$

Now try these:

$$6 + 4, 8 + 2, 9 + 4, 8 + 4, 5 + 6, 6 + 8, 7 + 5, 5 + 9$$

Using the same skills, try these multi-digit problems. Work left to right and come back to make trade-ins as needed. If you run into a blockade in carrying, you probably have trade-ins you haven't cleared.

$$28 + 65, 37 + 68, 147 + 326, 1793 + 2875$$

Adding decimal numbers is the same as adding whole numbers except you move the let the first several columns represent digits to the right of the decimal. Try these:

$$128.56 + 26.53, 375.21 + 638.75, 147.67 + 315.26, 17.93 + 528.75$$

You should also be able to do columns of numbers easily. Try these:

$$\begin{array}{r}
 2876 \\
 1295 \\
 9063 \\
 2920 \\
 + \underline{3995}
 \end{array}$$

(The answer is 20,149)

At this point you should be able to add any numbers you make up. Write them down, add them on the abacus, then check your work with a calculator. If you get good, you might try balancing your checkbook sometime.

### **Subtraction**

Subtraction is essentially undoing addition. Let's launch right in. Example:  $3-1$ .

Step 1: Set 3

Step 2: Take away 1

You know it is going to get more involved later on, but try these for starters:

$$2-1, 4-3, 8-2, 9-3$$

Example:  $6-2$ .

Step 1: Set 6

Step 2: You can't subtract 2, so subtract 5 then add back 3.

Try these:

$$5-1, 6-3, 8-4, 7-3$$

Example:  $12-9$

Step 1: Set 12

Step 2: Since there are not 9 units to take away, take away 10 and add back 1

Try these:

$$11-8, 13-9, 11-7$$

Example:  $10 - 5$

Step 1: Set 10

Step 2: Take away 10 and add back 5

Try these:

$11 - 5$ ,  $12 - 5$ ,  $13 - 4$

Example:  $11 - 2$

Step 1: Set 11

Step 2: Take away 10 and add back 8

Try these:

$10 - 3$ ,  $12 - 3$ ,  $12 - 4$

Example:  $15 - 7$

Step 1: Set 15

Step 2: Take away 10 and add back 3

Try these:

$15 - 9$ ,  $16 - 7$ ,  $18 - 9$

Example:  $11 - 6$

Step 1: Set 11

Step 2: Take away 10 and add back 4

Step 3: Trade in the five 1-beads for a 5-bead

The more advanced way to do this is:

Step 1: Set 11

Step 2: Take away 10 and try to add 4...

Step 3: ...to add 4, add 5 and take away 1

Try these:

$12 - 7$ ,  $13 - 8$ ,  $14 - 9$

Use what you have learned to do multi-digit problems. Again, remember to trade in beads whenever possible.

Try these:

$$92 - 35, 45 - 27, 85 - 44, 238 - 149$$

Make up more problems at random, write them down, do them with the abacus, and check them with a calculator.

If you would like to try out a simulated Soroban (Japanese Abacus) go to

[http://www.soroban.com/howto\\_abacus\\_eng.html](http://www.soroban.com/howto_abacus_eng.html)

(If you get their index page, select the Java applet.) The difference is you don't have the luxury of using the extra beads and trading them in later.

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<sup>1</sup> The gradual progression of problems is largely based on the sequence given in the manual accompanying the Lotus-Flower Brand abacusses (abaci?) purchased by the school.